SS 2017

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Manifolds and Group actions

Homework 8

Mandatory Exercise 1. (7 points) Let $A \in \mathfrak{sl}(2)$, the traceless matrices.

a) Show that there exists a $\lambda \in \mathbb{R} \cup i\mathbb{R}$ such that

$$\exp(A) = \cosh(\lambda)I + \frac{\sinh(\lambda)}{\lambda}A$$

and compute λ explicitly.

b) Show that that $\exp : \mathfrak{sl}(2) \to SL(2)$ is not surjective.

Mandatory Exercise 2. (8 Points)
Consider the set $H \subset GL(n)$ of matrices of the form

$$H = \{\begin{pmatrix} A & 0 \\ C & B \end{pmatrix} \mid A \in GL(k), B \in GL(n-k), C \in \mathrm{Mat}_{(n-k)\times k}(\mathbb{R})\}$$

a) Show that H is a closed Lie subgroup, and conclude that H acts freely and properly on $GL(n)$ from the left.

b) Give the dimension of the quotient manifold

$$Gr(n,k) = GL(n)/H,$$

which is called the Grassmannian.

c) Discuss how $Gr(n,k)$ describes the space of k-planes in $\mathbb{R}^n$.

Mandatory Exercise 3. (5 Points)
Consider

$$SL(2) = \{A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in GL(2) \mid \det A = 1\}.$$

Show that $SL(2)$ is diffeomorphic to $S^1 \times \mathbb{R}^2$. *Hint: Introduce new variables* $a = p+q, d = p-q, b = r+s$ *and* $c = r-s$

Hand in preferably on 19th of June in the pigeonhole on the third floor.